Exercises

Solving Linear Equations

Exercise 1. Solve the following systems of linear equations.

(a)
$$\begin{aligned} x_1 + 2x_2 &= 8 \\ 4x_1 + 3x_2 &= 17 \end{aligned}$$

(b)
$$\begin{aligned} 2x_1 + 4x_2 &= 17 \\ x_1 + 2x_2 &= 8 \end{aligned}$$

(c)
$$\begin{aligned} -3x_1 + x_2 - 2x_3 &= 4 \\ -6x_1 + 2x_2 - 4x_3 &= 8 \\ -15x_1 + 5x_2 - 10x_3 &= 20 \end{aligned}$$

(d)
$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 0 \\ 4x_1 + 5x_2 + 6x_3 &= 3 \\ 7x_1 + 8x_2 + 9x_3 &= 6 \end{aligned}$$

Exercise 2. Compute the inverse A^{-1} of the following matrix.

$$A = \begin{pmatrix} 2 & -2 & 1 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{pmatrix}$$

Exercise 3. Check if the following vectors are linear independent.

(a) $v_1^\top = (1, 2), \quad v_2^\top = (-1, -2)$

(b) $v_1^\top = (1, 3, 0, 2), \quad v_2^\top = (3, 9, 2, 8), \quad v_3^\top = (5, 10, 7, 12)$

Exercise 4. Solve the system of linear equations

$$\begin{aligned} 2x - ay + z &= 2 \\ x + 2y - z &= b, \end{aligned}$$

where $a, b \in \mathbb{R}$ are some constants.

Exercise 5. Compute the inverse X^{-1} of the matrix

$$X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

where $a, b, c, d \in \mathbb{R}$ with $ad \neq bc$. You may further assume without loss of generality that $a \neq 0$.

Exercise 6. Consider the following system of linear equations

$$\begin{aligned} x_1 + x_2 + x_3 &= 2q \\ 2x_1 - 3x_2 + 2x_3 &= 4q \\ 3x_1 - 2x_2 + px_3 &= q, \end{aligned}$$

where p and q are parameters. For which values of p, q is there
(i) an unique solution, (ii) no solution, (iii) infinitely many solutions?